ACOUSTICS

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McGRAW-HILL BOOK COMPANY

New York Toronto London

1954

traveled a distance $r - (b/2)\sin\theta$, and the sound pressure will be

$$p_1(r_1;t) = \frac{\sqrt{2}\,A_+}{r} e^{j\omega t} e^{-j(2\pi/\lambda)[r-(b/2)\sin\theta]} \tag{4.5a}$$

The wave from source 2 will have traveled a distance $r + (b/2)\sin\theta$, so that

$$p_2(r_2;t) = \frac{\sqrt{2}\,A_+}{r} e^{j\omega t} e^{-j(2\pi/\lambda)[r+(b/2)\sin\theta]} \tag{4.5b}$$

The sum of $p_1 + p_2$, assuming $r \gg b$, gives

$$p(r,t) = \frac{\sqrt{2}\,A_+}{r} e^{j\omega t} e^{-j(2\pi/\lambda)r} \left(e^{j(\pi b/\lambda)\sin\theta} + e^{-j(\pi b/\lambda)\sin\theta}\right) \tag{4.6}$$

Multiplication of the numerator and the denominator of Eq. (4.6) by

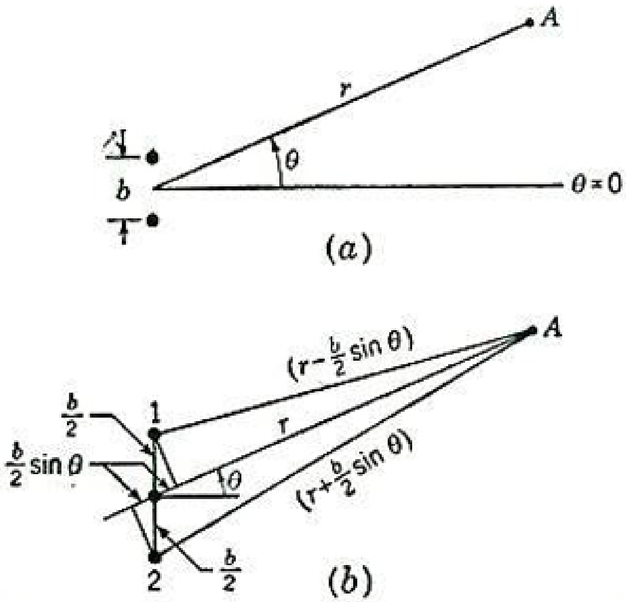

FIG. 4.2. Two simple (point) sources vibrating in phase located a distance b apart and at distance r and angle θ with respect to the point of measurement A.

$\exp(j\pi b \sin\theta/\lambda) - \exp(-j\pi b \sin\theta/\lambda)$ and replacement of the exponentials by sines, yields

$$p(r,t) = \frac{\sqrt{2}\,A_+}{r} e^{j\omega t} e^{-j(2\pi r/\lambda)} \frac{\sin[(2\pi b/\lambda)\sin\theta]}{\sin[(\pi b/\lambda)\sin\theta]} \tag{4.7}$$

The equation for the magnitude of the rms sound pressure $|p|$ is

$$|p| = \frac{2A_+}{r} \left| \frac{\sin[(2\pi b/\lambda)\sin\theta]}{2\sin[(\pi b/\lambda)\sin\theta]} \right| \tag{4.8}$$

The portion of this equation within the straight lines yields the directivity pattern.

Referring to Fig. 4.2, we see that if b is very small compared with a wavelength, the two sources essentially coalesce and the pressure at a distance r at any angle θ is double that for one source acting alone. The directivity pattern will be that of Fig. 4.1.

As b gets larger, however, the pressures arriving from the two sources will be different in phase and the directivity pattern will not be a circle.

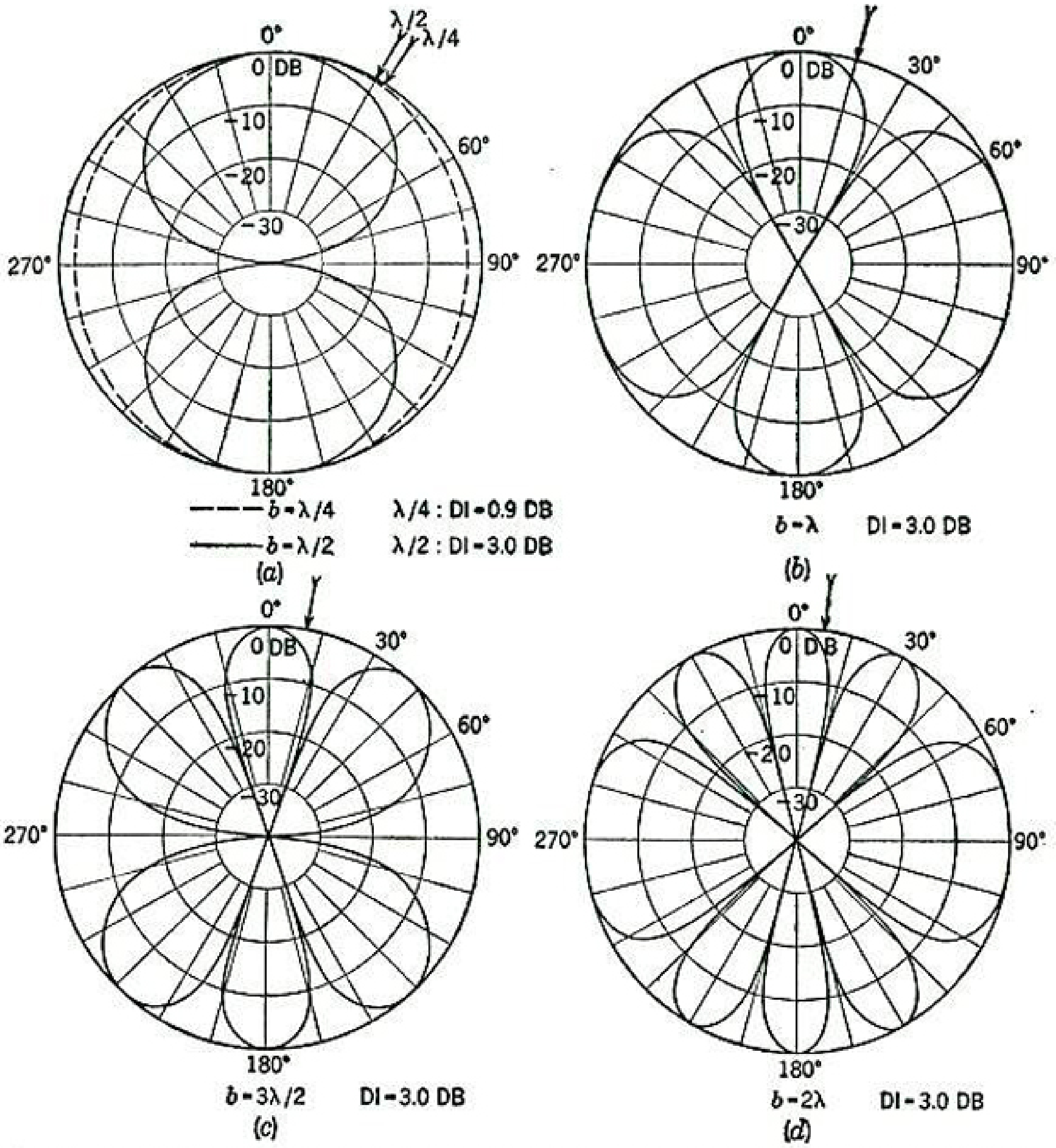

FIG. 4.3. Directivity patterns for the two simple in-phase sources of Fig. 4.2. Symmetry of the directivity patterns occurs about the axis passing through the two sources. Hence, only a single plane is necessary to describe the directivity characteristics at any particular frequency. The boxes give the directivity index at $\theta = 0°$. One angle of zero directivity index is also indicated. (The directivity index is discussed in Part XI.)

In other words, the sources will radiate sound in some directions better than in others. As a specific example, let $b = \lambda/2$. For $\theta = 0$ or $180°$ it is clear that the pressure arriving at a point A will be double that from either source. However, for $\theta = \pm 90°$ the time of travel between the whenever it decreases on the other. Hold the two transducers about 0.5 m apart with *both* diaphragms facing the floor (not facing each other). Let one transducer radiate a low-frequency sound and the other act as a microphone connected to the input of an audio amplifier. As we see from Fig. 4.7, no sound pressure will be produced at the diaphragm of the microphone, but there will be transverse particle velocity. A particle velocity is always the result of a pressure gradient in the direction of the velocity. Therefore, the diaphragm of the microphone will be caused to move when the two transducers are held as described above. When one of the transducers is rotated through 90° about the axis joining the units, the diaphragm of the microphone will not move because the pressure gradient will be in the plane of the diaphragm. Hence, the sound wave appears to be plane polarized.

You have now learned the elementary principles governing the directional characteristics of sound sources. We shall be able to use these

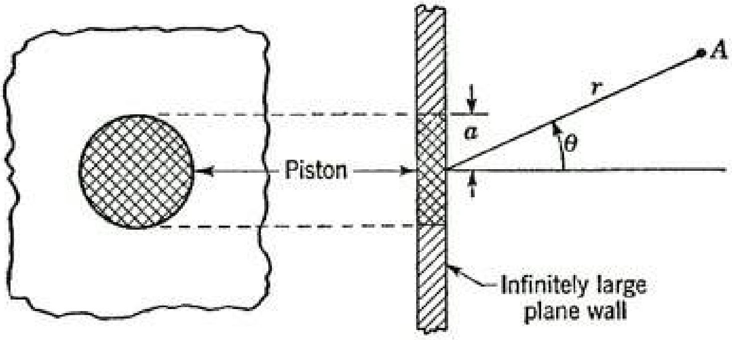

FIG. 4.9. Rigid circular piston in a rigid baffle. The point of measurement A is located at distance r and angle θ with respect to the center of the piston.

principles in understanding the measured or calculated behavior of some of the more complicated sound sources found in acoustics.

4.3. Plane Piston Sources. *Rigid Circular Piston in Infinite Baffle.* Many radiating sources can be represented by the simple concept of a vibrating piston located in an infinitely large rigid wall. The piston is assumed to be rigid so that all parts of its surface vibrate in phase and its velocity amplitude is independent of the mechanical or acoustic loading on its radiating surface. The rigid wall surrounding the piston is usually called a baffle, which, by definition, is a shielding structure or partition used to increase the effective length of the external transmission path between the front and back of the radiating surface.

The geometry of the problem is shown in Fig. 4.9. We wish to know the sound pressure at a point A located at a distance r and an angle θ from the center of the piston. To do this, we divide the surface of the piston into a number of small elements, each of which is a simple source vibrating in phase with all the other elements. The pressure at A is, then, the sum in magnitude and phase of the pressures from these elementary elements.

This summation appears in many texts[6] and, for the case of r large compared with the radius of the piston a, leads to the equation

$$p(r,t) = \frac{\sqrt{2}\, jf\rho_0 u_0 \pi a^2}{r} \left[\frac{2J_1(ka \sin\theta)}{ka \sin\theta} \right] e^{j\omega(t-r/c)} \tag{4.17}$$

where u_0 = rms velocity of the piston

$J_1(\;)$ = Bessel function of the first order for cylindrical coordinates[6]

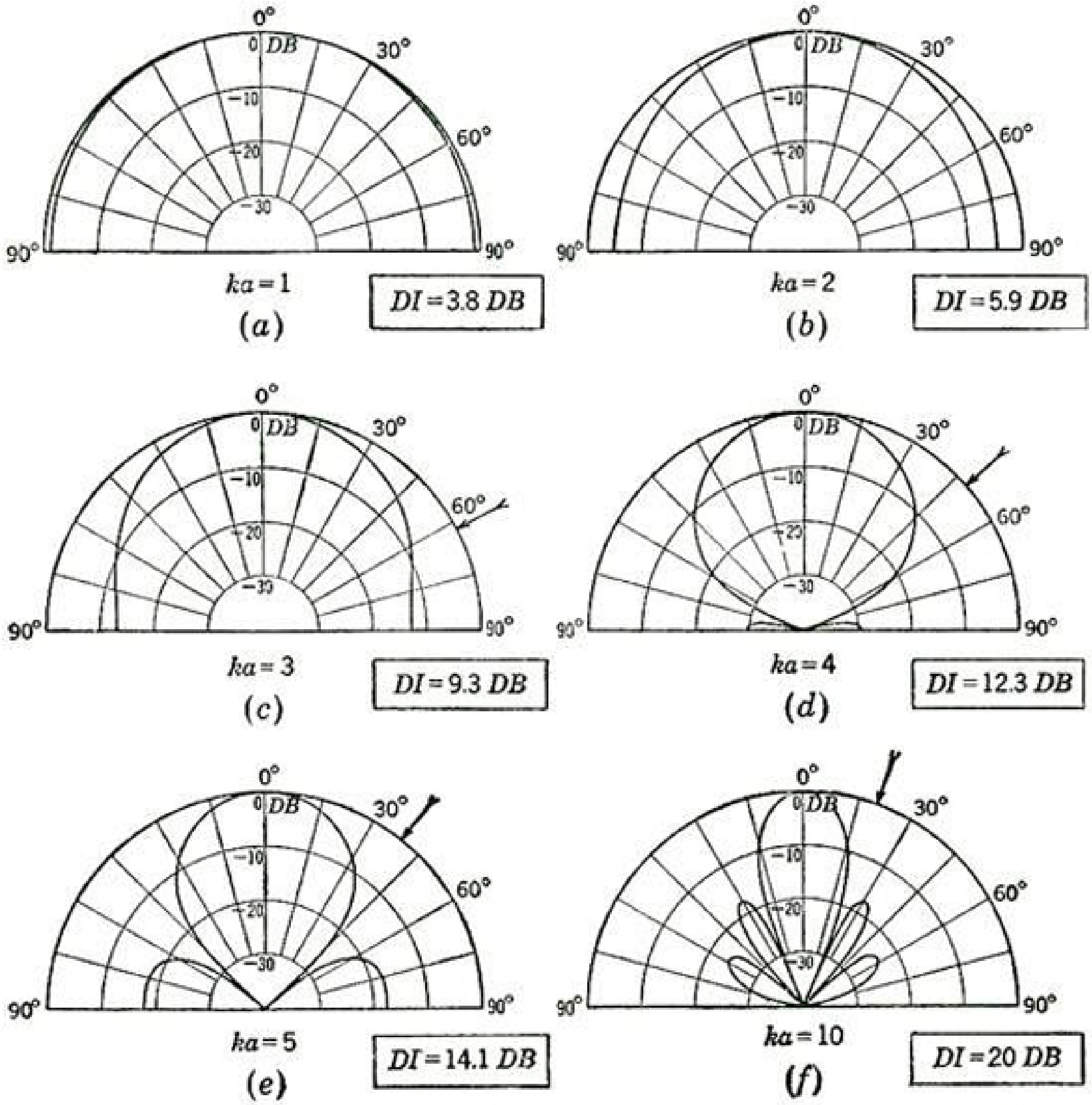

FIG. 4.10. Directivity patterns for a rigid circular piston in an infinite baffle as a function of $ka = 2\pi a/\lambda$, where a is the radius of the piston. The boxes give the directivity index at $\theta = 0°$. One angle of zero directivity index is also indicated. The DI never becomes less than 3 db because the piston radiates only into half-space.

The portion of Eq. (4.17) within the square brackets yields the directivity pattern and is plotted in decibels as a function of θ in Fig. 4.10 for six values of $ka = 2\pi a/\lambda$, that is, for six values of the ratio of the circumference of the piston to the wavelength.

When the circumference of the piston ($2\pi a$) is less than one-half wave-

[6] Morse, *op. cit.*, pp. 326–346. A table of Bessel functions is given on page 444.

length, that is, $ka < 0.5$, the piston behaves essentially like a point source. When ka becomes greater than 3, the piston is highly directional. We see from Fig. 4.24 that an ordinary loudspeaker also becomes quite directive at higher frequencies in much the same manner as does the vibrating piston.

Rigid Circular Piston in End of a Long Tube.[7] In many instances, sound is radiated from a diaphragm whose rear side is shielded from the front side by a box or a tube. If the box does not extend appreciably beyond the edges of the diaphragm, its performance may be estimated by comparison with that of a rigid piston placed in the end of a long tube.

The geometrical situation is shown in Fig. 4.11. The pressure at point A is again found by summing the pressures from a number of small elements on the surface of the piston, each acting as a simple source. The

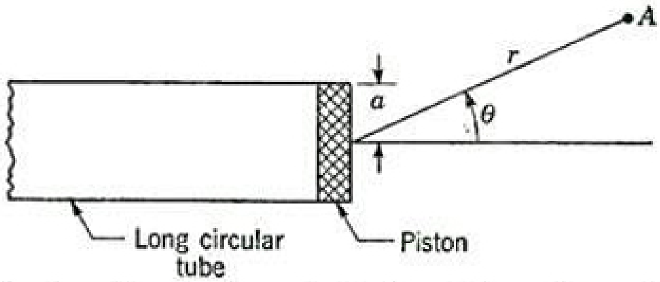

FIG. 4.11. Rigid circular piston in the end of a long tube. The point of measurement is located at distance r and angle θ with respect to the center of the piston.

solution of this problem is complex, however, because radiation can take place in all directions and the sound must diffract around the edge of the tube to get to the left-hand part of space (Fig. 4.11). Hence, a theory that includes the effects of diffraction must be used in solving the problem analytically.

The results from such a theory are shown in Fig. 4.12 for six values of ka. It is assumed here also that the distance r is large compared with a, so that the directivity pattern applies to the far-field.

Rigid Circular Piston without Baffle.[8] To complete the cases wherein pistons are commonly used, we present the results of theoretical studies on the directivity pattern of a rigid piston of radius a without any baffle, radiating into free space. These results are shown graphically in Fig. 4.13 for four values of ka. It is interesting to note the resemblance between these curves and those for an acoustic doublet. In fact, to a first approximation, an unbaffled thin piston is simply a doublet, because an axial movement in one direction compresses the air on one side of it and causes a rarefaction of the air on the other side.

[7] H. Levine and J. Schwinger, On the Radiation of Sound from an Unflanged Circular Pipe, *Phys. Rev.*, **73**: 383–406 (Feb. 15, 1948).

[8] F. M. Wiener, On the Relation between the Sound Fields Radiated and Diffracted by Plane Obstacles, *J. Acoust. Soc. Amer.*, **23**: 697–700 (1951).

$R_M^2 \gg X_M^2$, the sound pressure increases linearly with frequency f. This condition is shown in Fig. 7.6 by the dashed line.

High Frequencies. Referring back to Fig. 7.4*a*, we see that there is a possibility of a second resonance taking place involving L/B^2l^2 and the masses $M_{MD} + X_{MR}/\omega$. The voice-coil velocity at this resonance can be determined from the circuit of Fig. 7.5*d*. The resonance frequency will occur when

$$\frac{\omega L(B^2l^2)}{\omega^2L^2 + (R_g + R_E)^2} = \omega M_{MD} + 2X_{MR} \tag{7.16}$$

We must note, however, that if $(R_g + R_E)^2$ is large compared with $L^2\omega^2$, the reactance of the capacitance L/B^2l^2 and resistance $B^2l^2/(R_g + R_E)$

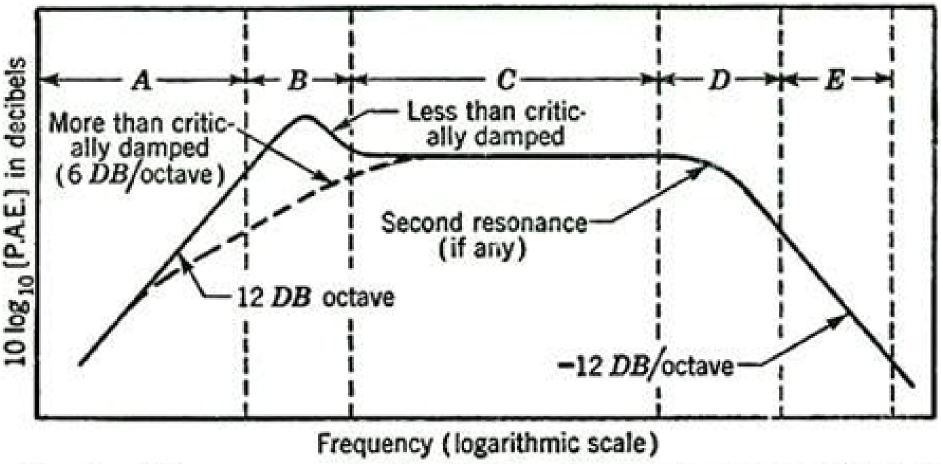

FIG. 7.6. Graph of the power available efficiency in decibels of a hypothetical direct-radiator loudspeaker in an infinite baffle. It is assumed that the diaphragm acts like a rigid piston over the entire frequency range. The power is the total radiated from *both* sides of the diaphragm. Zero decibels is the reference power available efficiency level. The solid curve is for a loudspeaker with a Q_T of about 2. The dashed curve is for a Q_T equal to about 0.5.

in parallel becomes that of a negative inductance equal to $-B^2l^2L/(R_g + R_E)^2$. In this case, no resonance can occur.

The solution of Fig. 7.5*d* applies to the peak of the region marked D in Fig. 7.6.

At frequencies above the second resonance frequency, the radiation resistance on each side of the diaphragm becomes approximately equal to $\pi a^2\rho_0c$, where a is the effective radius of the loudspeaker. Also, ω^2M_{MD} becomes large compared with the resistance in the circuit, and ω^2L^2 becomes large compared with $(R_g + R_E)^2$. The voice-coil velocity is determined from Fig. 7.5*e*. The power available efficiency is

$$\text{PAE} = \frac{800R_gB^2l^2\rho_0c\pi a^2}{\omega^4L^2{M_{MD}}^2} \tag{7.17}$$

This region is marked E in Fig. 7.6. Here, the power output decreases by 12 db for each doubling of frequency.

The response curve given in Fig. 7.6 is for a typical loudspeaker used for the reproduction of music in the home. For this application, the mass of the cone is made as light as possible and the compliance of the suspension as high as possible consistent with mechanical stability. For special applications, C_{MS} can be small so that the resonance frequency is high. Also, it is common in practice to make R_M so large that the velocity u_c is nearly constant as a function of frequency through regions B and C. In this case, the sound pressure increases linearly with frequency, and there is no flat region C.

7.8. Reference Efficiency. It is convenient to define a reference efficiency which permits one to plot the shape of the frequency-response curve without showing the actual acoustic power that is being radiated at the time. The reference power available efficiency (both sides of the diaphragm) is defined as,

$$\text{PAE}_{\text{ref}} = \frac{800 R_g B^2 l^2 \Re_{MR}}{(R_g + R_E)^2 \omega^2 (M_{MD} + 2M_{M1})^2} \tag{7.18}$$

or, with the help of Eq. (7.9),

$$\text{PAE}_{\text{ref}} = \frac{800 R_g B^2 l^2 S_D{}^2 \rho_0}{2\pi c (R_g + R_E)^2 (M_{MD} + 2M_{M1})^2} \tag{7.19}$$

If the loudspeaker is less than critically damped, Eq. (7.19) gives the actual response in frequency region C, which lies above the first resonance frequency. Even for loudspeakers that are highly damped so that there is no flat region C, Eq. (7.19) forms a convenient reference to which the rest of the curve is compared.

Expressed as a ratio, the PAE response at medium and low frequencies where the radiation is nondirectional [see Eq. (7.15)] is

$$\frac{\text{PAE}}{\text{PAE}_{\text{ref}}} = \frac{\omega^2 (M_{MD} + 2M_{M1})^2}{R_M{}^2 + X_M{}^2} \tag{7.20}$$

At the resonance frequency ω_0, where $X_M = 0$,

$$\frac{\text{PAE}}{\text{PAE}_{\text{ref}}} = \frac{\omega_0{}^2 (M_{MD} + 2M_{M1})^2}{R_M{}^2} \equiv Q_T{}^2 \tag{7.21}$$

where Q_T is analogous to the Q of electrical circuits. Equations (7.20) and (7.21) may be expressed in decibels by taking $10 \log_{10}$ of both sides of the equations.

In Chap. 8 of this book, on Loudspeaker Enclosures, design charts are presented from which it is possible to determine, without laborious computation, the sound pressure from a direct-radiator loudspeaker as a function of frequency including the directivity characteristics. Methods for determining the constants of loudspeakers and of box and bass-reflex enclosures are also presented. If the reader is interested only in learning

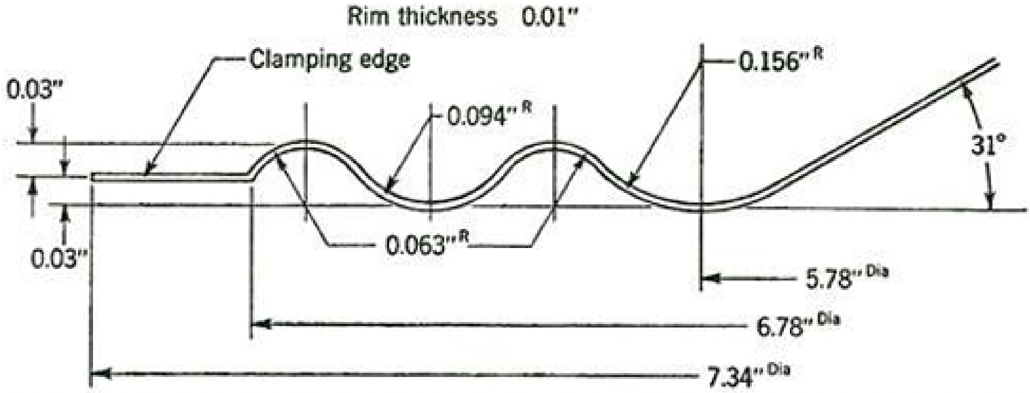

FIG. 7.7. Detail of the edge of a felted-paper loudspeaker cone from an 8-in. loudspeaker. [*After Corrington, Amplitude and Phase Measurements on Loudspeaker Cones, Proc. IRE,* **39**: 1021–1026 (1951).]

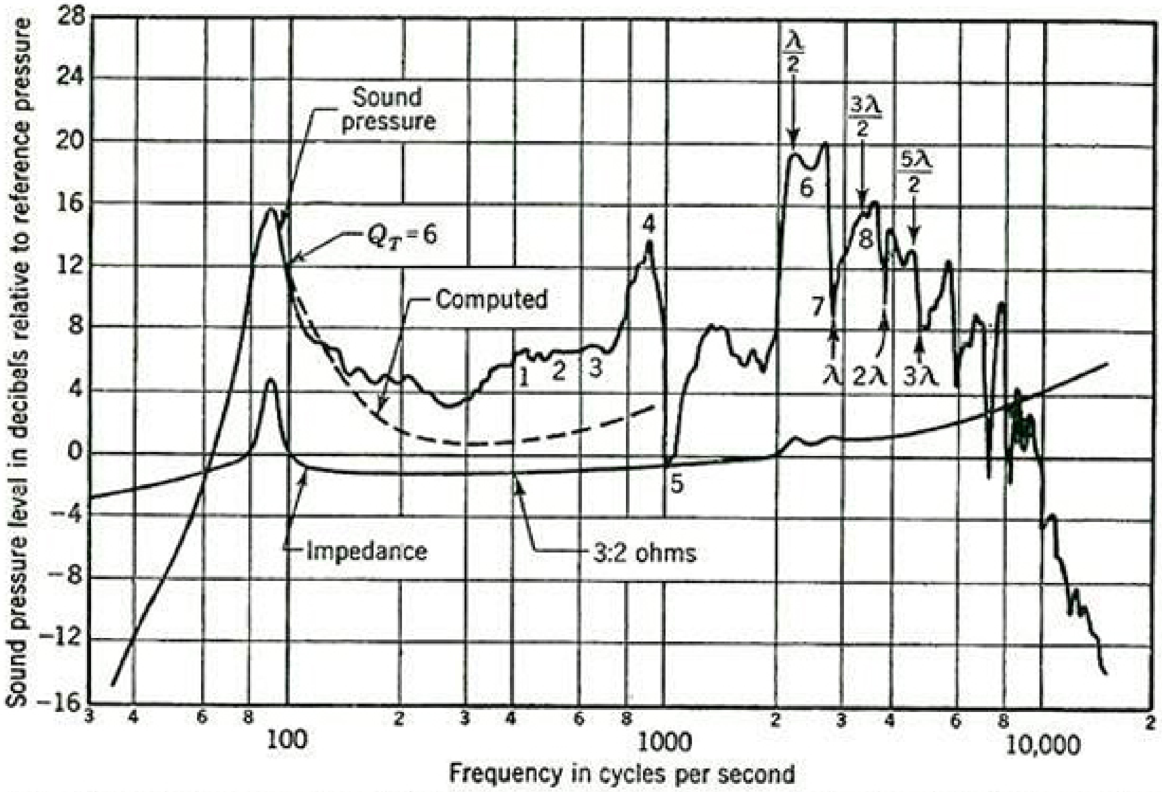

FIG. 7.8. Relative power-available response of an 8-in.-diameter loudspeaker mounted in an infinite baffle. The dashed curve was computed from Figs. 8.12 and 8.13 for $Q_T = 6$. [*After Corrington, Amplitude and Phase Measurements on Loudspeaker Cones, Proc. IRE,* **39**: 1021–1026 (1951).]

from Eqs. (7.28), (7.31), and (7.32), that $R_E = R_g$ and

$$M_{MC} = M'_{MD} + 2M_{M1}.$$

It is not usual, however, that the voice coil should be this massive, for the reason that a large voice coil demands a correspondingly large magnet structure.

Values of voice-coil resistances and masses for typical American loudspeakers are given in Table 8.1 of the next chapter.

7.11. Diaphragm Behavior. The simple theory using the method of equivalent circuits, which we have just derived, is not valid above some frequency between 300 and 1000 cps. In the higher frequency range the cone no longer moves as a single unit, and the diaphragm mass M_{MD} and also the radiation impedance change. These changes may occur with great rapidity as a function of frequency. As a result, no tractable mathematical treatment is available by which the exact performance of a loudspeaker can be predicted in the higher frequency range.

A detailed study of one particular loudspeaker is reported here as an example of the behavior of the diaphragm.[2] The diaphragm is a felted paper cone, about 6.7 in. in effective diameter (see Fig. 7.7), having an included angle of 118°.

The sound-pressure-level response curve for this loudspeaker measured on the principal axis is shown in Fig. 7.8. This particular loudspeaker has, in addition to its fundamental resonance, other peaks and dips in the response at points 1 to 8 as indicated on the curve.

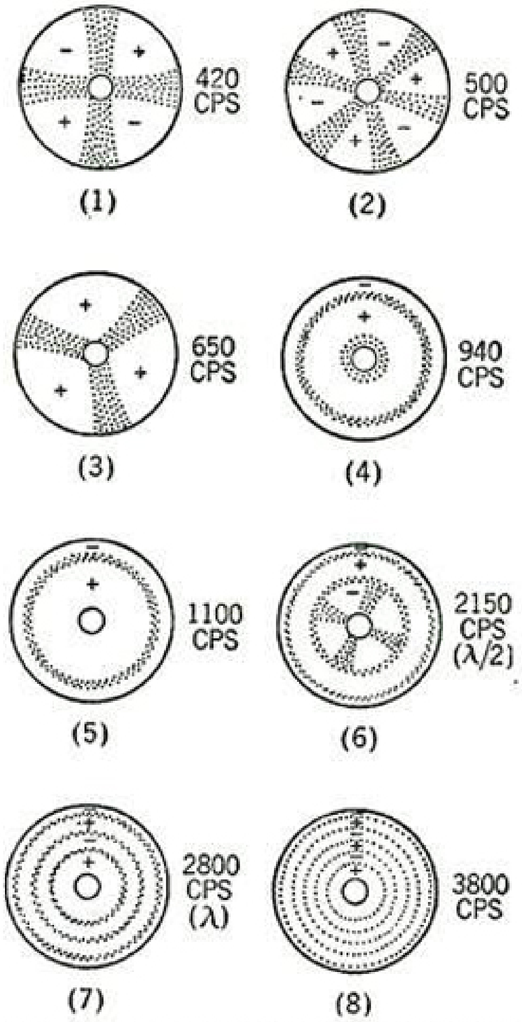

FIG. 7.9. Nodal pattern of the cone of the loudspeaker whose response curve is given in Fig. 7.8. The shaded and dashed lines indicate lines of small amplitude of vibration. The (+) and (−) signs indicate regions moving in opposite directions, *i.e.*, opposite phases. [*After Corrington with changes.*]

The major resonance at 90 cps is the principal resonance and has the relative amplitude given by Eq. (7.21). Above that is the fairly flat region that we have called region *C*. At point 1, which is located at 420 cps, the cone breaks up into a resonance of the form shown by the first sketch in Fig. 7.9. Here, there are four nodal lines on the cone extending radially, and four regions of maximum movement. As indicated by the plus and minus signs, two regions move outward while two regions move inward. The net effect is a pumping of air back and forth across the nodal lines. The cone is also vibrating as a whole in and out of the page. The net change in the output is an increase of about 5 db relative to that computed. A similar situation exists at point 2 at

[2] M. S. Corrington, Amplitude and Phase Measurements on Loudspeaker Cones, *Proc. IRE*, **39**: 1021–1026 (1951).

Reference Volume Velocity and Sound Pressure. A reference diaphragm volume velocity is arbitrarily defined here by the equation

$$|U_c|_C = \frac{e_g Bl}{(R_g + R_E)\omega M_A S_D} \tag{8.26}$$

This reference volume velocity is equal to the actual volume velocity above the resonance frequency under the special condition that R_A^2 of Eq. (8.19) is small compared with $\omega^2 M_A^2$. This reference volume velocity is consistent with the reference power available efficiency defined in Par. 7.8.

The reference sound pressure at low frequencies, where it can be assumed that there is unity directivity factor, is found from Eqs. (8.16) and (8.26).†

$$|p|_C = \frac{e_g Bl \rho_0}{(R_g + R_E) M_A 4\pi r S_D} \tag{8.27}$$

It is emphasized that the reference sound pressure will not be the actual sound pressure in the region above the resonance frequency unless the motion of the diaphragm is mass-controlled and unless the directivity factor is nearly unity. The reference pressure is, however, a convenient way of locating "zero" decibels on a relative sound-pressure-level response curve, and this is the reason for defining it here.

Radiated Sound Pressure for $ka < 1$. The radiated sound pressure in the frequency region where the circumference of the diaphragm ($2\pi a$) is less than a wavelength (*i.e.*, where there is negligible directivity) is found by taking the ratio of Eq. (8.16) to Eq. (8.27), using Eq. (8.25) for $|U_c|$.

$$\left|\frac{p}{p_C}\right| = \frac{\omega/\omega_0}{\left[\dfrac{1}{Q_T^2} + \left(\dfrac{\omega}{\omega_0} - \dfrac{\omega_0}{\omega}\right)^2\right]^{1/2}} \tag{8.28}$$

The ratio, in decibels, of the sound pressure at the resonance frequency ω_0 to the reference sound pressure is

$$20 \log_{10} \left|\frac{p}{p_C}\right|_0 = 20 \log_{10} Q_T \tag{8.29}$$

For *flat* response down to the lowest frequency possible, Q_T should approximately equal unity. (Note that for critical damping $Q_T = 0.5$.)

Referring back to Eq. (7.39), we find that we suggested for satisfactory transient response that $R_M/M_M = R_A/M_A > 184\ \text{sec}^{-1}$. Let us see what this means in terms of Q_T.

† Equation (8.27) is the same as that derived in Example 7.1, p. 194 except for a factor of 2 in the denominator. This factor of 2 expresses the difference between radiation into full space as compared to radiation into half space (infinite baffle case).

a total mass M_{MD}, a mechanical compliance C_{MS}, and a mechanical resistance $R_{MS} = 1/r_{MS}$. The quantity r_{MS} is the mechanical responsiveness of the diaphragm in mohms (mobility ohms).

Behind the diaphragm is a space that is usually filled with a soft acoustical material. At low frequencies this space acts as a compliance C_{MD} which can be lumped in with the compliance of the diaphragm. At high frequencies the reactance of this space becomes small so that the space behind the diaphragm becomes a mechanical radiation resistance $R_{MB} = 1/r_{MB}$ with a magnitude also equal to that given in Eq. (9.1). This resistance combines with the mechanical radiation resistance of the throat, and the diaphragm must develop power both to its front and its back. Obviously, any power developed behind the diaphragm is wasted, and at high frequencies this sometimes becomes as much as one-half of the total generated acoustic power.

In front of the diaphragm there is an air space with compliance C_{M1}. At low frequencies the air in this space behaves like an incompressible fluid, that is, ωC_{M1} is small, and all the air displaced by the diaphragm passes into the throat of the horn. At high frequencies the mechanical reactance of this air space becomes sufficiently low (*i.e.*, the air becomes compressible) so that all the air displaced by the diaphragm does not pass into the throat of the horn.

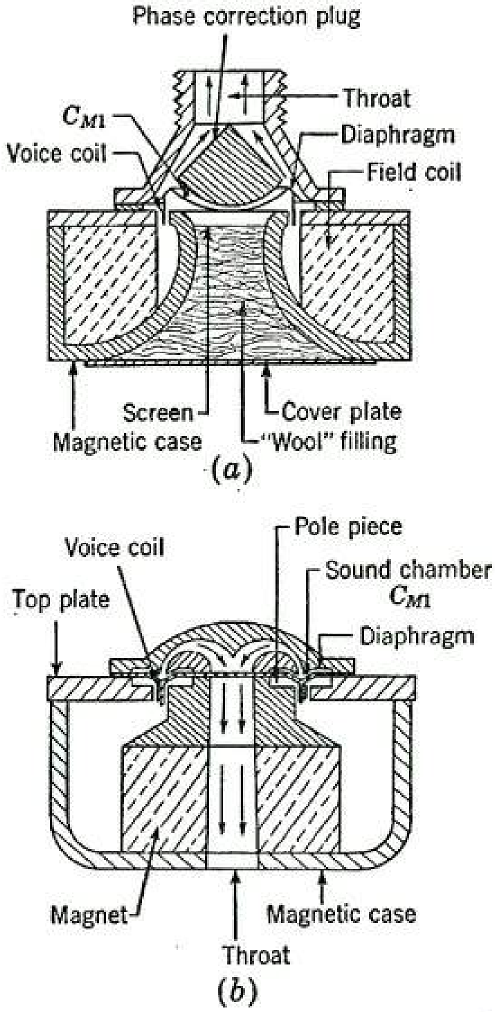

FIG. 9.2. Cross section of two typical horn driving units. The diaphragm couples to the throat of the horn through a small cavity with a mechanical compliance C_{M1}.

The voice coil has an electrical resistance R_E and inductance L. As stated above, z_{MT} is the mechanical mobility at the throat of the horn.

By inspection, we draw the mobility-type analogous circuit shown in Fig. 9.3. In this circuit forces "flow" *through* the elements, and the velocity "drops" *across* them. The generator open-circuit voltage and resistance are e_g and R_g. The electric current is i; the linear velocity of the voice coil and diaphragm is u_c; the linear velocity of the air at the throat of the horn is u_T; and the force at the throat of the horn is f_T. As before, the area of the diaphragm is S_D, and that of the throat is S_T.

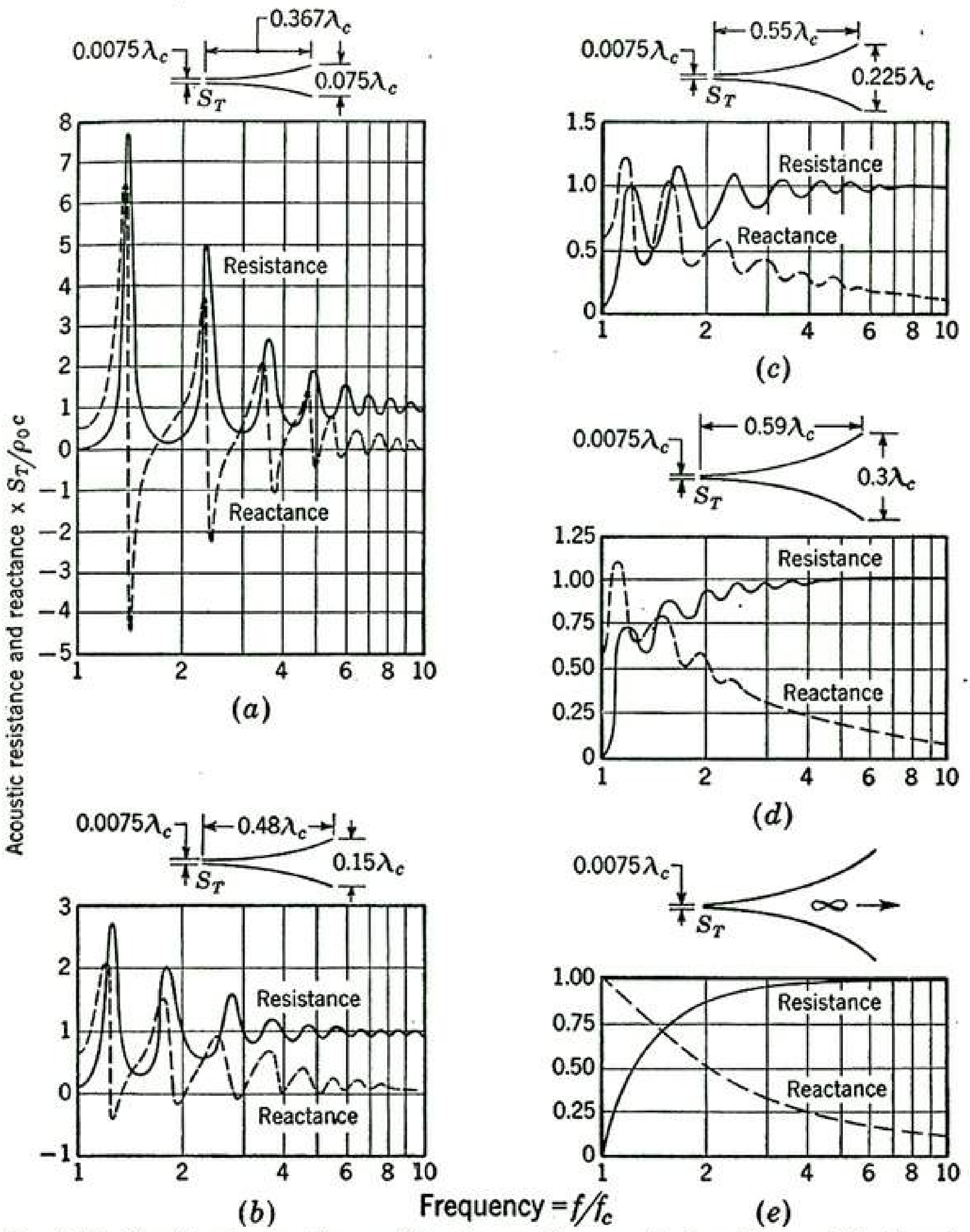

FIG. 9.10. Graphs showing the variation in specific acoustic impedance at the throat of four exponential horns as a function of frequency with bell diameter as the parameter. The cutoff frequency $f_c = mc/4\pi$ and the throat diameter $= 0.0075\ c/f_c$; both are held constant. Bell circumferences are (a) $C = 0.236\lambda_c$, (b) $C = 0.47\lambda_c$, (c) $C = 0.71\lambda_c$, (d) $C = 0.94\lambda_c$, and (e) $C = \infty$. (*After Olson.*)

Even when the source is nominally nondirectional, the directivity factor Q depends on the position of the source in the room. For example, we see from Fig. 4.20 that if a nondirectional source is located in a plane wall, $Q = 2$ because the power is radiated into hemispherical space only.

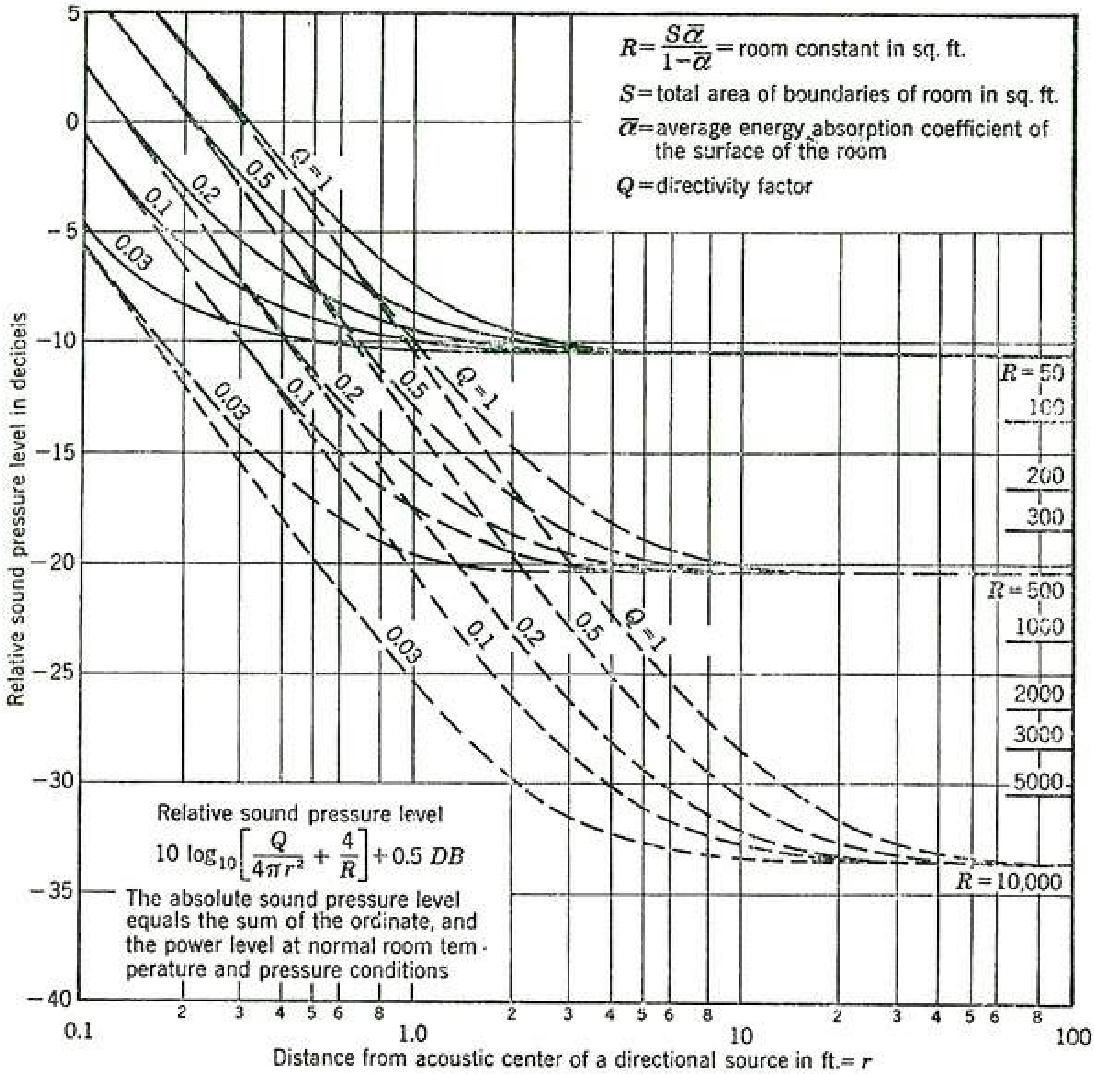

FIG. 10.23. Same as Fig. 10.19, except for a directional sound source with a directivity factor Q less than unity.

In Table 10.2, we show values of Q for four typical positions of a small nondirectional source in a large room.

TABLE 10.2. Values of Q for Small Nondirectional Sources Located at Typical Positions in a Large Rectangular Room

Position in room	Q
Near center	1
In center of one wall	2
At edge, halfway between floor and ceiling	4
At corner	8

We mentioned earlier that when the enclosure is small, it will react on the source to modify its radiated power. Whether an enclosure is to be